PHYSICAL REVIEW B **82**, 041309(R) (2010)

Prediction of resonant all-electric spin pumping with spin-orbit coupling

Valentina Brosco,[1,2,3] Markus Jerger,[4] Pablo San-José,[3] Gergely Zarand,[5,6] Alexander Shnirman,[7,6] and Gerd Schön[3,6]
[1]*Dipartimento di Fisica, Università "La Sapienza," P.le A. Moro 2, 00185 Roma, Italy*
[2]*ISC-CNR, Via dei Taurini 19, 00185 Roma, Italy*
[3]*Institut für Theoretische Festkörperphysik, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*
[4]*Physikalisches Institut, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*
[5]*Theoretical Physics Department, Institute of Physics, Budapest University of Technology and Economy, Budapest H-1521, Hungary*
[6]*DFG, Center for Functional Nanostructures (CFN), Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*
[7]*Institut für Theorie der Kondensierten Materie, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*
(Received 4 January 2010; revised manuscript received 2 July 2010; published 28 July 2010)

All-electric devices for the generation and filtering of spin currents are of crucial importance for spintronics experiments and applications. Here we consider a quantum dot with spin-orbit interaction coupled to two metallic leads. After analyzing, the conditions for having nonvanishing spin currents in an adiabatically driven two-terminal device, we focus on a dot with two resonant orbitals and we show by specific examples that both spin filtering and pure spin current generation can be achieved. Finally, we discuss the effect of the Coulomb interaction.

DOI: 10.1103/PhysRevB.82.041309 PACS number(s): 72.10.−d, 85.35.Be, 85.75.−d

A lot of theoretical and experimental effort has been devoted in recent years to spintronics, i.e., the design and control of spin-based electronic devices.[1] As a result, it became possible to inject and filter spin-polarized currents,[2] to detect spin accumulation,[3–5] and to produce ferromagnetic spin valves.[6] While an all-electrical control of spin currents would be clearly advantageous so far most theoretical and experimental designs involve ferromagnetic leads or require the application of external magnetic fields.

Adiabatic pumping of charge in a cyclically modulated potential was first proposed by Thouless[7] and later studied in a variety of mesoscopic devices.[8–12] More recently, pumping of spins in nanostructures has been proposed as well, again with most mechanisms relying on external magnetic or exchange fields,[13–18] or the presence of ferromagnetic leads.[19] Indeed, in one experiment spin pumping in a magnetic field has been observed.[20] On the other hand, as first discussed in Refs. 16 and 17, it is also possible to pump spin through quantum wires in the absence of external magnetic fields provided that spin and orbit are coupled. However, changing the spin-orbit (SO) coupling, as proposed in Ref. 16 is a rather difficult task, while the chaotic cavity investigated in Ref. 17 remains to a large extent uncontrolled.

In the present paper we show that, in the presence of SO coupling, resonances associated with avoided level crossings of a quantum dot can be exploited to *pump* spin in a controlled way purely by cycling electrical gates. We shall focus on quantum dots with parameters such that the level spacing of the dot exceeds the typical width of the levels, $\delta\epsilon > \Gamma$. In this parameter regime individual states act as resonances with positions and couplings to the external leads tunable by external gates.[21,22] *Resonant spin pumping* emerges, when two of the levels lie close to the Fermi energy so that the SO coupling mixes them. We will show that in the vicinity of such resonant avoided level crossings the quantum dot can be used as an all-electric spin battery and pumping cycles with transmitted spin of order $\hbar/2$ per cycle can be constructed. We also show that it is possible to choose the cycle parameters such that no charge is transferred through the quantum dot.

We consider an elastic scatterer coupled to two quasi-one-dimensional leads. In the left lead, far from the scattering region we can define longitudinal charge and spin current operators, J_L^0 and J_L^i, as

$$J_L^{\mu}(z,t) = -\frac{i}{2m}\int dxdy[\psi_L^{\dagger}\sigma^{\mu}(\partial_z\psi_L) - \text{H.c.}], \qquad (1)$$

where σ^0 is the unit matrix while σ^{μ} with $\mu=1,2,3$ denote the usual Pauli matrices. The spinor field $\psi_L(x,y,z,t)$ destroys an electron in the left lead and ∂_z denotes the partial derivative with respect to the coordinate along the wire.

At low temperatures, we can use the approach of Brower,[9,17] to express the spin ($\vec{S}_L$) and charge (Q_L) pumped into the left lead within an adiabatic cycle as

$$Q_L = -\frac{e}{2\pi}\int_0^T \text{Im}\left[\text{Tr}\left\{(\Lambda_L \otimes \sigma_0)\frac{d\mathcal{S}}{dt}\mathcal{S}^{\dagger}\right\}\right]dt, \qquad (2a)$$

$$\vec{S}_L = -\frac{\hbar}{2\pi}\int_0^T \text{Im}\left[\text{Tr}\left\{(\Lambda_L \otimes \vec{\sigma})\frac{d\mathcal{S}}{dt}\mathcal{S}^{\dagger}\right\}\right]dt. \qquad (2b)$$

Here $\mathcal{S}=\mathcal{S}(E_F,t)$ is the instantaneous scattering matrix at the Fermi energy and Λ_L stands for the projector onto the left channel.

Following a strategy similar to Avron *et al.*,[10] we decompose the scattering matrix to identify physical processes which contribute to spin and charge pumping. The presence of time-reversal symmetry implies that, in an appropriate basis, the scattering matrix is self-dual,[23] i.e., $\mathcal{S}=\sigma_y\mathcal{S}^T\sigma_y$, and, for a quantum dot connecting two one-mode leads, $\mathcal{S}$ can be decomposed as

$$\mathcal{S} = U^0 U T U^{\dagger} U^0, \qquad (3)$$

where the matrices U, U^0, and T are defined as follows:

 ©2010 The American Physical Society

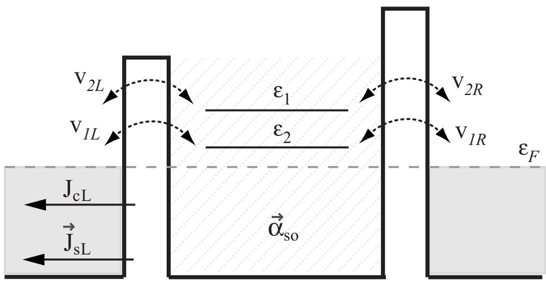

FIG. 1. Level structure of the quantum dot. Due to the spin orbit field, $\vec{\alpha}_{so}$, pumping leads to charge and spin current.

$$U^0=\begin{pmatrix} e^{i\phi_L} & 0\\ 0 & e^{i\phi_R}\end{pmatrix}\otimes\sigma_0,\quad U=\begin{pmatrix}\mathcal{U}_L & 0\\ 0 & \mathcal{U}_R\end{pmatrix},$$

$$T=\begin{pmatrix} -\sqrt{1-T_0} & \sqrt{T_0}\\ \sqrt{T_0} & \sqrt{1-T_0}\end{pmatrix}\otimes\sigma_0. \quad (4)$$

The constant T_0 denotes the transmission coefficient of the dot and $\mathcal{U}_{L,R}$ are two-dimensional unitary matrices.

Substituting Eqs. (3) and (4) into Eqs. (2a) and (2b) we obtain

$$Q_L=\frac{e}{2\pi}\int_0^T[(1-T_0)(\dot{\phi}_R-\dot{\phi}_L)]dt, \quad (5a)$$

$$\vec{S}_L=\frac{i\hbar}{2\pi}\int_0^T T_0\mathrm{Tr}\{(\mathcal{U}_L^\dagger\vec{\sigma}\mathcal{U}_L)(\mathcal{U}_L^\dagger\dot{\mathcal{U}}_L-\mathcal{U}_R^\dagger\dot{\mathcal{U}}_R)\}dt. \quad (5b)$$

In Eq. (5a) we dropped a term $(e/2\pi)\int(\dot{\phi}_R+\dot{\phi}_L)dt$, representing the charge accumulated on the scatterer, which vanishes over a complete cycle. The first term in Eq. (5a) is finite even for $T_0=0$. This limit corresponds to peristaltic pumping, whereby a charge is first moved from the left lead to the dot while the right contact is kept closed and subsequently from the dot to the right lead with the left contact kept closed.

Interestingly, the pumped spin given by Eq. (5b) is proportional to T_0. Therefore peristaltic spin pumping is not possible in the presence of time-reversal symmetry. Also, while over a full cycle the charge is conserved, $Q_L+Q_R=0$, the spin is, in general, not conserved: in particular if $[\mathcal{U}_L,\mathcal{U}_R]\neq 0$ we have $\vec{S}_R+\vec{S}_L\neq 0$. This is not surprising since spin-orbit coupling—ultimately responsible for the spin pumping—allows for the transfer of angular momentum to the underlying lattice. We remark that the pumped spin in Eq. (5b) transforms as a vector under spin rotations in the left lead while it is invariant under spin rotation in the right lead, as it was also clear from the definition in Eq. (1).

To calculate explicitly the charge and spin transferred through a dot in the vicinity of an avoided level crossing, let us now introduce a noninteracting model to describe a dot with two orbital levels, $|n\sigma\rangle$ $(n=1,2,\sigma=\pm)$ as shown in Fig. 1. In the presence of (SO) coupling the isolated dot is described by the Hamiltonian $H^d=\Sigma d_{n\sigma}^\dagger\mathcal{H}^d_{n\sigma,n'\sigma'}d_{n'\sigma'}$ with the operators $d_{n\sigma}^\dagger$ creating an electron in state $|n\sigma\rangle$ and the 4×4 matrix $\mathcal{H}^d$ given by

$$\mathcal{H}^d=\begin{pmatrix}\varepsilon_1\sigma^0 & -i\vec{\alpha}\cdot\vec{\sigma}\\ i\vec{\alpha}\cdot\vec{\sigma} & \varepsilon_2\sigma^0\end{pmatrix}. \quad (6)$$

Here ε_n indicate the energies of the dot orbital states measured from the Fermi energy and the real vector $\vec{\alpha}$ is the SO field. Choosing the spin quantization axis parallel to $\vec{\alpha}$, we can express the full Hamiltonian as $H=\Sigma_\sigma H_\sigma$ with

$$H_\sigma=\sum_n\varepsilon_n d_{n\sigma}^\dagger d_{n\sigma}+i\alpha s_\sigma(d_{1\sigma}d_{2\sigma}^\dagger-\mathrm{H.c.}) + \sum_{n,\xi,\lambda}(v_n^\lambda c_{\xi\sigma\lambda}^\dagger d_{n\sigma}+\mathrm{H.c.})+\sum_{\xi,\lambda}\xi c_{\xi\sigma\lambda}^\dagger c_{\xi\sigma\lambda} \quad (7)$$

and with $s_\sigma=\pm 1$ for spin parallel/antiparallel to $\vec{\alpha}$ and $\alpha=|\vec{\alpha}|$. Here $c_{\xi\sigma\lambda}^\dagger$ creates a conduction electron of energy ξ and spin σ in lead λ. In the following, we shall assume that while the levels ε_n and the tunneling amplitudes v_n^λ can be tuned via gate voltages, $\vec{\alpha}$ remains approximately constant over a pumping cycle. Choosing $\vec{\alpha}$ as a quantization axis we can write the scattering matrix in spin-diagonal form, $\mathcal{S}=e^{i(\phi_L+\phi_R)}\mathcal{S}_\uparrow\oplus\mathcal{S}_\downarrow$, with S_σ parametrized using $\phi=\phi_L-\phi_R$ as

$$\mathcal{S}_\sigma=\begin{pmatrix}-e^{i\phi}\sqrt{1-T_0} & e^{is_\sigma\theta}\sqrt{T_0}\\ e^{-is_\sigma\theta}\sqrt{T_0} & e^{-i\phi}\sqrt{1-T_0}\end{pmatrix}. \quad (8)$$

Obviously, the phase ϕ is related to charge pumping while the phase θ determines the amount of spin pumped into the leads. If we only modify two external parameters, r_1 and r_2, during a cycle, we can use Stokes theorem to recast the pumped charge and spin as

$$Q_L=-\frac{e}{2\pi}\int\int \mathrm{d}^2 r B_c^{r_1r_2}, \quad (9)$$

$$\vec{S}_L=\hat{\alpha}\frac{\hbar}{4\pi}\int\int\mathrm{d}^2 r B_s^{r_1r_2}, \quad (10)$$

where the charge and spin Brouwer's fields are defined as $B_c^{r_1r_2}=\partial_{r_1}T_0\partial_{r_2}\phi-\partial_{r_2}T_0\partial_{r_1}\phi$ and $B_s^{r_1r_2}=\partial_{r_1}T_0\partial_{r_2}\theta-\partial_{r_2}T_0\partial_{r_1}\theta$, respectively, and $\hat{\alpha}=\vec{\alpha}/\alpha$.

To calculate the pumped charge and spin via Eqs. (9) and (10), we need to relate the transmission T_0 and the angles ϕ and θ to the pumping parameters, which are typically either the tunnel couplings or the positions of the dot levels, appearing in the Hamiltonian H. This can be done most easily by expressing $\mathcal{S}$ in terms of the Green's function G^d of the dot[24,25] $\mathcal{S}_\sigma^{\lambda\lambda'}=\delta_{\lambda\lambda'}-2\pi i\sqrt{\rho_\lambda\rho_{\lambda'}}\Sigma_{n,n'}v_n^\lambda v_{n'}^{\lambda'*}G^d_{n,n'\sigma}(\varepsilon_F)$, $[G^d(\omega)]^{-1}_{nn',\sigma}=\omega\delta_{nn'}-(\mathcal{H}^d_\sigma)_{nn'}+i\pi\Sigma_\lambda\rho_\lambda v_n^{\lambda*}v_{n'}^\lambda$, where ρ_λ is the density of states in lead λ at the Fermi energy. From these equations we can obtain T_0, ϕ, and θ in terms of the bare parameters of the dot. Introducing $w=v_{1L}v_{2R}-v_{2L}v_{1R}$ we can express ϕ and θ as follows:

$$\tan(\theta)=\frac{\alpha w}{\varepsilon_1 v_{2L}v_{2R}+\varepsilon_2 v_{1L}v_{1R}}, \quad (11)$$

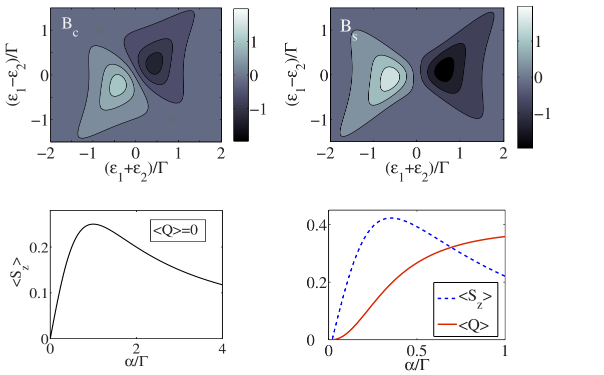

FIG. 2. (Color online) Upper Panels: Brouwer's charge and spin field, $B_c^{\varepsilon_1,\varepsilon_2}$ and $B_s^{\varepsilon_1,\varepsilon_2}$, in the $[(\varepsilon_1-\varepsilon_2),(\varepsilon_1+\varepsilon_2)]$ plane for $v_{2L}=-v_{1R}=2v_{2R}=1.25v_{1L}=\sqrt{\Gamma/(2\pi\rho)}$, $\alpha/\Gamma=0.25$. Lower Panels: pumped spin through a symmetric dot (left) and pumped spin and charge through a nonsymmetric dot (right) as a function of the SO coupling α for a triangular cycle: $(\varepsilon_1+\varepsilon_2)\in[0,10\alpha]$ and $(\varepsilon_1-\varepsilon_2)\in[-(\varepsilon_1+\varepsilon_2),\varepsilon_1+\varepsilon_2]$. In the symmetric dot case the tunnel couplings are $v_{2L}=-v_{1R}=v_{2R}=v_{1L}=\sqrt{\Gamma/(2\pi\rho)}$ while in the nonsymmetric dot case they are the same as those of the upper panel.

$$\tan(\phi)=\pi\rho\frac{\varepsilon_1(v_{2L}^2-v_{2R}^2)+\varepsilon_2(v_{1L}^2-v_{1R}^2)}{\varepsilon_1\varepsilon_2-\alpha^2+\pi^2\rho^2w^2}. \tag{12}$$

Clearly, both levels are involved in spin pumping: if one of the levels is decoupled from the leads then $w=0$ and only charge pumping is possible. Coupling the first and second level to the two leads with equal amplitudes also leads to a vanishing of the spin current.

In the following we analyze two kinds of cycles, "orbital energy cycles" and "tunnel coupling cycles," where, respectively, only the orbital energies of the levels are varied or also the tunnel coupling to the leads. Spin filtering and pure spin pumping can be achieved in both cycles.

For orbital energy cycles the relevant components of the Brouwer's fields are $B_s^{\varepsilon_1,\varepsilon_2}$ and $B_c^{\varepsilon_1,\varepsilon_2}$. For weak SO coupling these components have a dipolar structure in the plane $(\varepsilon_1,\varepsilon_2)$, as shown in the upper panels of Fig. 2. As for the single-level dot, studied in Ref. 26, the maxima of the charge field correspond to points of resonant transmission. The resonances of the spin field are instead located along the line $\varepsilon_1^r+\varepsilon_2^r=0$ where the renormalized energies, ε_1^r and ε_2^r, are defined as $\varepsilon_1^r=\varepsilon_1 v_{2L}v_{2R}/w$ and $\varepsilon_2^r=\varepsilon_2 v_{1L}v_{1R}/w$. As we will show, the structure of the fields and the amount of pumped charge and spin depend very sensitively on the ratio between tunneling amplitudes and SO coupling. To start with, let us consider the special case of a *symmetric dot* with a symmetric and an antisymmetric orbital. In this case the Hamiltonian commutes with the operator $\pi\equiv\Sigma\otimes P$, where Σ and P denote the spin inversion and parity operators, and the tunneling amplitudes satisfy the following relations: $v_{1L}=v_{1R}=v_1$ and $v_{2L}=-v_{2R}=v_2$. In this symmetrical situation, a variation in the level energies, ε_1 and ε_2, generates a *pure spin current* independently of the details of the cycle. In fact, as already noted by Aleiner *et al.*,[27] parity imposes additional constraints on the scattering matrix that along with time-reversal symmetry lead to the vanishing of Brouwer's charge field. In Fig. 2 (lower left panel) we plot the total spin pumped through a symmetric dot with $v_1=v_2=\sqrt{\Gamma/(2\pi\rho)}$ for a triangular pumping cycle around the infinite area, $|\varepsilon_1-\varepsilon_2|\le|\varepsilon_1+\varepsilon_2|$ and $0\le|\varepsilon_1+\varepsilon_2|<\infty$. As one can see, the pumped spin depends nonmonotonously on the ratio α/Γ, the maximum occurring at $\alpha/\Gamma=1$. In the lower right panel of Fig. 2 we show a more realistic situation: We plot the spin and charge pumped through a nonsymmetric dot in a triangular cycle bounded by the lines, $|\varepsilon_1-\varepsilon_2|\le|\varepsilon_1+\varepsilon_2|$ and $0\le|\varepsilon_1+\varepsilon_2|\le 10\Gamma$. In this case both the pumped charge and spin are nonvanishing and depend nonmonotonously on the SO coupling.

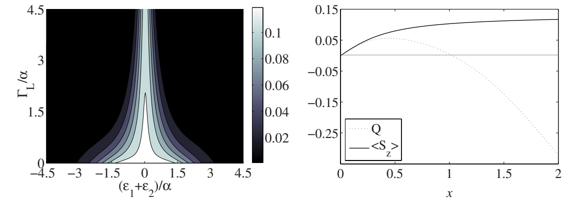

FIG. 3. (Color online) Left Panel: Brouwer's spin field $B_s^{\Gamma_L,(\varepsilon_1+\varepsilon_2)}$ in the $[\Gamma_L,(\varepsilon_1+\varepsilon_2)]$-plane for $\Gamma_{kM}=c_{kM}^2\Gamma_M$ with $k=1,2$ and $M=L,R$, $c_{1L}=c_{2L}=0.8$, $c_{1R}=0.7$, and $c_{2R}=0.6$. The two levels are assumed degenerate, $\varepsilon_1-\varepsilon_2=0$, and the coupling to the right lead is $\Gamma_R/\alpha=3$. Right Panel: pumped spin and charge for a rectangular cycle $\Gamma_L/\alpha\in[0,5]$ and $(\varepsilon_1+\varepsilon_2)/\alpha\in[0,x]$ as a function of x.

In the experiments it may be difficult to vary independently the energies of the two levels. For this reason we now consider pumping cycles where only the tunneling rates to the left lead and the offset of the two levels with respect to the Fermi energy in the leads are varied. Specifically, we express the tunneling amplitudes as follows, $v_{kM}=c_{kM}\sqrt{\Gamma_M/(2\pi\rho)}$ with $k\in[1,2]$ and $M\in[L,R]$ and we assume that only Γ_L can be varied using some external gate. The factors c_{kM}, describing the overlap between the kth dot's level and the scattering states in lead M, are assumed to be constant during the cycle. In Fig. 3 (left panel) we plot Brouwer's spin field $B_s^{\Gamma_L,\varepsilon_1+\varepsilon_2}$. In this case, again, Brouwer's charge field is nonvanishing, however, with an appropriate tuning of the cycle shape, we can have pure spin currents. This is shown in Fig. 3 (right panel) where we plot the total charge and spin pumped for a rectangular cycle, $\Gamma_L/\alpha\in[0,5]$ and $(\varepsilon_1+\varepsilon_2)/\alpha\in[0,x]$ as a function of x.

To observe resonant spin pumping, the width of the two levels involved needs to be smaller or in the range of the level spacing. In this regime interaction effects, neglected so far, become important, and they lead to a Coulomb blockade if all levels are far away from the resonances, and to the formation of a mixed valence state close to resonance. However, in all cases, the ground state of the dot is a Fermi liquid, and therefore low-temperature spin pumping can be described in terms of a scattering matrix, just as for the noninteracting system.[28–30] The only difference is that the parameters of this scattering matrix depend in a very nontrivial way on the bare model parameters. Determining the precise functional dependence of the scattering matrix on the gate voltages and the level positions is a cumbersome task beyond the scope of the present paper. The qualitative structure of

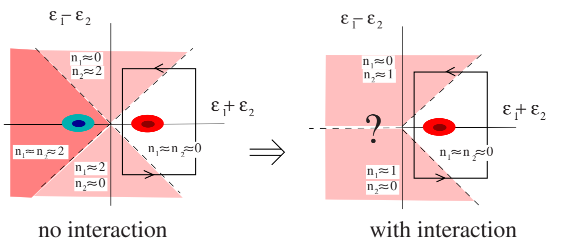

FIG. 4. (Color online) Sketch of the effect of interactions. In the regime $\epsilon_1, \epsilon_2 > 0$, where the dot occupancy is small the noninteracting theory applies.

the "phase diagram" and the spin pumping field is sketched in Fig. 4 for a dot with large Coulomb interaction. While the Coulomb interaction suppresses double occupancy for $\epsilon_1, \epsilon_2 < 0$, its effects are expected to be not crucial in the regime, $\epsilon_1, \epsilon_2 > 0$, where the levels are only partially occupied. There noninteracting theory is expected to work reasonably well and the resonant spin pumping discussed here should be observable. Note that all the results presented so far and shown in Figs. 2 and 3 concern cycles performed in the "weakly interacting region," $\epsilon_1, \epsilon_2 > 0$.

The situation discussed so far is somewhat ideal. Besides Coulomb interaction several issues need to be carefully analyzed in the experimental implementation of a resonant spin pump. First, in order to preserve spin coherence, pumping has to be faster than spin relaxation in the dot, along with adiabaticity, which requires the pumping period to be longer than the electrons' dwell time, τ_d ($\sim 1/\Gamma$ in our case), this condition leads to rather strict constraints on the pumping frequency, namely, $\Gamma_{so} < \omega < 1/\tau_d$. The latter inequality essentially sets a limit on the maximum amount of spin and charge pumped in a cycle. Moreover, since the structure of the Brouwer's fields is essentially determined by the spin-orbit coupling α, for our protocols to work the pumping parameters need to be controlled on a scale on the order of α. Eventually, in order to have accurate pumping, the device should be kept at low temperatures such that $k_B T < 1/\tau_d$. Good candidates for the implementations of our scheme are thus quantum dots with a sizable spin-orbit coupling to have a robust pumping and a low spin-relaxation rate. At low temperatures, in most dots spin relaxation is mainly due to hyperfine coupling[31] but this mechanism can be quenched through an appropriate choice of the material. Spin-relaxation times on the order of tens of nanoseconds[32] were obtained in p-doped quantum dots but even hole spin relaxation times on the order of 1 ms have been reported in case of well-isolated p-type quantum dots.[33] On the other hand, pumping with a period of 100 ps can be already considered adiabatic in a dot with an escape rate of the order of 0.5 meV. In this case, at a temperature of 100 mK, currents on the order of 50–100 pA are achievable with a spin-orbit coupling on the order of the escape rate.

In conclusion, we have studied spin and charge pumping in a quantum dot with spin-orbit interaction coupled to two metallic leads. Using Brouwer's scattering approach to pumping, we first analyzed the general restrictions imposed by time-reversal symmetry on the pumped current. We then focused on the case of a dot with two resonant levels and showed that a spin on the order of $\hbar/2$ can be pumped in a cycle. Due to the very simple structure of Brouwer's fields, our pumping scheme is ideally accurate and robust respect to small variations in the cycle shape. Finally we discussed the effect of Coulomb interaction, which should introduce only inessential corrections in the regime of partial dot occupation.

ACKNOWLEDGMENTS

We were supported by the EU FP7 under Grant Agreement No. 238345 (GEOMDISS), the DFG-Schwerpunktprogramm 1285 "Halbleiter-Spintronik," and under the Hungarian Proposals OTKA No. K73361 and No. OMFB-0029.

[1] S. A. Wolf *et al.*, Science **294**, 1488 (2001).
[2] F. J. Jedema *et al.*, Nature (London) **410**, 345 (2001); K. Uchida *et al.*, *ibid.* **455**, 778 (2008).
[3] R. M. Potok *et al.*, Phys. Rev. Lett. **89**, 266602 (2002).
[4] X. Lou *et al.*, Nat. Phys. **3**, 197 (2007); P. Kotissek *et al.*, *ibid.* **3**, 872 (2007).
[5] S. M. Frolov *et al.*, Nature (London) **458**, 868 (2009).
[6] J. C. Sankey *et al.*, Nat. Phys. **4**, 67 (2008).
[7] D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).
[8] M. Büttiker *et al.*, Z. Phys. B **94**, 133 (1994).
[9] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).
[10] J. E. Avron *et al.*, Phys. Rev. B **62**, R10618 (2000).
[11] Y. Makhlin and A. D. Mirlin, Phys. Rev. Lett. **87**, 276803 (2001).
[12] M. Moskalets and M. Büttiker, Phys. Rev. B **68**, 075303 (2003).
[13] E. R. Mucciolo *et al.*, Phys. Rev. Lett. **89**, 146802 (2002).
[14] T. Aono, Phys. Rev. B **67**, 155303 (2003).
[15] R. Citro and F. Romeo, Phys. Rev. B **73**, 233304 (2006).
[16] M. Governale *et al.*, Phys. Rev. B **68**, 155324 (2003).
[17] P. Sharma and P. W. Brouwer, Phys. Rev. Lett. **91**, 166801 (2003).
[18] A. Schiller and A. Silva, Phys. Rev. B **77**, 045330 (2008).
[19] J. Splettstoesser *et al.*, Phys. Rev. B **77**, 195320 (2008).
[20] S. K. Watson *et al.*, Phys. Rev. Lett. **91**, 258301 (2003).
[21] K. MacLean *et al.*, Phys. Rev. Lett. **98**, 036802 (2007).
[22] D. R. Yafaev, *Mathematical Scattering Theory* (AMS, Providence, RI, 1992).
[23] C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
[24] I. L. Aleiner and V. I. Falko, Phys. Rev. Lett. **87**, 256801 (2001).
[25] T. K. Ng and P. A. Lee, Phys. Rev. Lett. **61**, 1768 (1988).
[26] Y. Levinson *et al.*, Physica A **302**, 335 (2001).
[27] I. L. Aleiner *et al.*, Phys. Rev. B **62**, 10373 (2000).
[28] P. Nozieres and A. Blandin, J. Phys. (France) Lett. **41**, 193 (1980).
[29] M. Pustilnik and L. I. Glazman, Phys. Rev. Lett. **87**, 216601 (2001).
[30] P. Mehta *et al.*, Phys. Rev. B **72**, 014430 (2005).
[31] P. San-Jose *et al.*, Phys. Rev. Lett. **97**, 076803 (2006).
[32] B. Eble *et al.*, Phys. Rev. Lett. **102**, 146601 (2009).
[33] B. D. Gerardot *et al.*, Nature (London) **451**, 441 (2008).